

The Subjet Multiplicity in Quark and Gluon Jets

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Abstract

We calculate the mean number of subjets in quark and gluon jets in the final state of e^+e^- annihilation. Since “quark” and “gluon” jets are scheme-dependent objects, we stress the importance of using the same definition as in experimental analyses. We define the jets using the k_\perp algorithm at a coarse scale y_1 , and the subjets using a finer scale y_0 , in the same algorithm. Gluon jets are anti-tagged by the presence of heavy quarks in both other jets. Our result is exact to leading order in α_s , and resums leading and next-to-leading logarithmic terms in the ratio y_1/y_0 to all orders in α_s .

1 Introduction

Leading logarithmic QCD predicts that the multiplicity of hadrons in a gluon jet should be larger than that in a quark jet by the ratio of their colour charges, $C_A/C_F = 9/4$. Thus one would naïvely expect the multiplicity in three-jet e^+e^- events to be $(2C_F+C_A)/2C_F = 17/8$ times higher than in two-jet events. Experimentally, however this ratio is close to unity[1]. The sub-leading corrections of relative size $\sqrt{\alpha_s}$ and even α_s [2], are known for the case of jets produced by a colour singlet source. Although they reduce this ratio, they are small at the energies of modern experiments, and certainly not capable of bringing it down to unity. When the jets are produced in a three-jet ensemble, rather than by a colour singlet, the equivalent corrections can be much larger[3], but without assumptions about the non-perturbative hadronization process, it is impossible to calculate their size.

Considerable light was shed on this problem by the paper of Catani *et. al.*[4], who considered an analogous problem that can be solved purely perturbatively, namely the multiplicity of *subjets* in a jet. A jet algorithm is run twice with different cutoff scales, Q_0 and Q_1 , and the number of jets found at each, $\mathcal{M}_{0,1}$. \mathcal{M}_0 is then called the number of subjets in an \mathcal{M}_1 -jet event. By considering the region $\Lambda_{\text{QCD}} \ll Q_0 \ll Q_1$, they were able to study the region in which logarithmic terms dominate, as in the hadron multiplicity problem, but which is fully under perturbative control. They found that colour coherence during the formation of the three-jet state was responsible for a depletion in the multiplicity in three-jet events relative to the naïve, incoherent, expectation. Experimental results[5] have shown good agreement with the predictions of [4] for not too small Q_0 , although there is clear evidence of an additional suppression in three-jet events as one approaches the non-perturbative region.

More recently, using their excellent ability to tag heavy quark jets, experiments at LEP have been able to measure the subjet multiplicity in individual jets that are anti-tagged as the gluon jet in three-jet events[6]. This allows a more direct measurement of the multiplicity ratio between gluon and quark jets in three-jet events. Although the results were in good agreement with Monte Carlo event generators that incorporate coherence[7, 8, 9], it was not possible to compare with the results of [4] for two reasons. Firstly, their method is not able to keep track of which jet the registered subjets are part of, it simply counts the total number of subjets. Secondly, the experiments found that to obtain a sufficient lever-arm in Q_1/Q_0 for the logarithmic behaviour to become important before the non-perturbative behaviour, Q_1 had to be rather large $\sim Q$, where Q is the total e^+e^- annihilation energy, while [4] work in the $Q_1 \ll Q$ limit.

In this paper, we correct both deficiencies by numerically integrating an expression that exactly reproduces both the leading order expression in α_s and the next-to-leading order resummation of logarithms of Q_1/Q_0 , but not logarithms of Q/Q_1 . Our result is therefore uniformly reliable for all Q_1/Q_0 , and not too small Q_1 . In section 2, we give details of the event definition, which is matched as closely as possible to the experimental one. In section 3, we discuss the logarithmic behaviour and define the numerical treatment, which allows us to resum logarithms of Q_1/Q_0 , without losing the exact treatment of the $Q_1 \sim Q$ region. We compare our results with the data of [6] and find good agreement where the hadronization corrections are expected to be small. Finally in section 4 we briefly discuss the dependence on the energies of the individual jets within the three-jet sample.

2 The Event Definition

In order to be able resum logarithms of the jet resolution scale, jets must be defined using an algorithm such as the k_\perp algorithm[10] in which the phase-space for multiple emission factorizes in the same way as the matrix elements. This allows jet multiplicities to be resummed to at least next-to-leading logarithmic accuracy to all orders in α_s [4, 11].

The algorithm defines for every pair of particles in the final state a ‘closeness’ measure,

$$y_{ij} \equiv \frac{k_{\perp ij}^2}{Q^2} \equiv \frac{2 \min(E_i, E_j)^2 (1 - \cos \theta_{ij})}{Q^2}. \quad (1)$$

If the smallest value of y_{ij} is below a cutoff y_{cut} , the pair i, j is merged into a single pseudoparticle with momentum

$$p_{ij} = p_i + p_j, \quad (2)$$

and the process is repeated, with pseudoparticles treated on an equal footing with particles. If all y_{ij} are above y_{cut} the algorithm is terminated and all remaining particles and pseudoparticles are called jets.

To define subjets, the algorithm is first run with a cutoff $y_{\text{cut}} = y_0$, and the resulting particles and pseudoparticles are called subjets. The algorithm is then continued with a cutoff $y_{\text{cut}} = y_1 > y_0$, and the resulting particles and pseudoparticles are called jets. During this second stage of clustering, the fate of each subjet is kept track of, giving the number of subjets ‘inside’ each jet.

At present, the exact matrix elements for the final state of e^+e^- annihilation are only known up to $\mathcal{O}(\alpha_s^2)$ [12]. Since the multiplicity of subjets in an n -jet event only becomes non-trivial at $\mathcal{O}(\alpha_s^{(n-1)})$, we are confined to $n \leq 3$ in exact calculations. Furthermore, since the two-jet final state does not contain gluon jets, we are only interested in $n \geq 3$. Therefore, we can only calculate a single non-trivial term, the leading order expression for the subjet multiplicity in three-jet events. This can be easily found by numerically integrating the exact matrix element within the phase-space region given by the event definition.

So far, we have fully defined the subjet structure of the event, but to obtain the maximum amount of information from such events, we must also define the flavour structure. Namely which jets are quark jets and which are gluon jets. Beyond leading logarithmic order, the results will depend on this definition, so we must use the same definition as in experimental analyses. This is done using heavy flavour tagging: in three-jet events in which two of the jets contain heavy quarks, the third is called a gluon jet. Then, using the subjet multiplicity in three-jet events, \mathcal{M}_3 , and in gluon jets, \mathcal{M}_g , the subjet multiplicity in quark jets can be found by assuming that every three-jet event consists of two quark jets and a gluon jet,

$$\mathcal{M}_q \equiv \frac{1}{2}(\mathcal{M}_3 - \mathcal{M}_g). \quad (3)$$

Using this definition, \mathcal{M}_q is a physical quantity, since both \mathcal{M}_3 and \mathcal{M}_g are.

At $\mathcal{O}(\alpha_s^2)$ there are two sub-processes to consider: $q\bar{q}gg$ and $q\bar{q}q^{(\prime)}\bar{q}^{(\prime)}$. For each case, one must consider what flavour to call a jet consisting of a pair of partons of given flavours. When one of the partons is a gluon, this is straightforward: the flavour of the jet is the flavour of the other parton. However if both partons are quarks or antiquarks, one must carefully consider how the flavour-tagging would assign the jet, recalling that

a gluon jet is defined as the untagged one in an event in which both the other jets are tagged as quark jets.

In the case of $q\bar{q}gg$, the combination $q + \bar{q}$ should be called a quark jet, since neither of the others is a quark.

In the case of the $q\bar{q}q^{(\prime)}\bar{q}^{(\prime)}$ final state, $q + \bar{q}$ is called a gluon jet, since both the others are quarks. For the combinations $q + q^{(\prime)}$ and $q + \bar{q}'$, the outcome will depend on the exact experimental procedure and the corrections that are applied to it. We make the simplifying assumption that each jet is equally likely to be tagged, so these are shared between gluon and quark jets in the ratio 1 to 2. These contributions make such a small contribution to the total cross-section that varying this assumption does not affect the final result significantly.

It is straightforward to integrate the leading order matrix element according to these definitions, and we obtain the results shown later, in figure 1.

3 Resumming Large Logarithmic Terms

When the ratio of cutoffs, y_1/y_0 , becomes large, double-logarithmic terms appear in the perturbative expansion, $\mathcal{M} \sim \alpha_s^n \log^{2n} y_1/y_0$. Such terms must be resummed to all orders in α_s to give a reliable prediction. Although this was done in [4], they used the simplifying limit $y_1 \ll 1$, and were not able to keep track of which jet contained which subjet. In this section we correct both deficiencies, by using a very similar trick to that first used in [13] for the subjet multiplicity in hadron collisions.

To illustrate the general method, we first work in logarithmic approximation, valid for $y_0 \ll y_1 \ll 1$. We use the usual notation $L_{0,1} = -\log y_{0,1}$. A three-jet configuration in which the gluon makes an angle θ with the nearer quark can increase the subjet multiplicity in five ways: gluon emission from the opposite quark, from the nearer quark at angles larger than θ , angles smaller than θ , emission from the gluon, and gluon splitting to quarks. To next-to-leading logarithmic accuracy, one can neglect recoils in the emission, and the multiplicity can be easily found,

$$\begin{aligned} \mathcal{M}_3 - 3 &= \frac{1}{2}\bar{\alpha}_s(L_0 - L_1)\left\{C_F(L_0 + L_1 - 3) + C_F\left(\frac{2}{3}L_1 - 1\right) + C_F\left(L_0 + \frac{1}{3}L_1 - 2\right)\right. \\ &\quad \left.+ C_A\left(L_0 - \frac{1}{3}L_1 - \frac{14}{3}\right) + \frac{2}{3}N_f\right\}, \end{aligned} \quad (4)$$

where $\bar{\alpha}_s = \alpha_s/2\pi$. The separate terms correspond to the different contributions just mentioned, and add up to the expression of [4].

Since there is one set of terms proportional to the colour charge of a quark, and another to the colour charge of a gluon, it is tempting to assume that these are the subjet multiplicities in the quark and gluon jets respectively. However, simple kinematics shows that this is not the case. For example, a gluon emitted from the nearer quark at a larger angle than θ could be merged with either the gluon or the quark, depending on the relative azimuths. There are non-trivial azimuthal correlations in the soft limit of the $q\bar{q}gg$ matrix element, which are not retained in the usual evolution equations because they average to zero. Therefore the resummed result of [4] *cannot* predict the number of subjets that are merged into each jet, although it can predict the total number of them. We shall shortly see how this problem can be solved without having to define new azimuth-dependent evolution equations.

Similarly, emission from the gluon can be merged with the quarks, depending on the azimuth and opening angle. However, simple kinematics also shows that emission from any jet at angles smaller than $\theta/2$ will always be merged with that jet. We make use of this fact by introducing an *arbitrary* parameter μ that separates the intrajet and interjet regions. We consider μ to be fixed and in the range $Q_0 \leq \mu \leq Q_1/2^*$, although in general one could make it a function of the three-jet kinematics. We then call emission from a jet of energy E at angles smaller than μ/E intrajet emission (note that θ must be larger than Q_1/E), and at angles larger than μ/E interjet emission. This gives us multiplicities

$$\mathcal{N}_{\text{intra}} - 3 = \frac{1}{2}\bar{\alpha}_s \left\{ 2C_F(L_{\mu 0}^2 - 3L_{\mu 0}) + C_A(L_{\mu 0}^2 - \frac{11}{3}L_{\mu 0}) + \frac{2}{3}N_f L_{\mu 0} \right\}, \quad (5)$$

where $L_{\mu 0} = \log \mu^2/Q_0^2$, and

$$\begin{aligned} \mathcal{N}_{\text{inter}} = & \frac{1}{2}\bar{\alpha}_s \left\{ 2C_F[L_{\mu 0}(2L_1 + 2L_{\mu 1}) + L_{\mu 1}(2L_1 + L_{\mu 1} - 3)] \right. \\ & \left. + C_A[L_{\mu 0}(\frac{2}{3}L_1 + 2L_{\mu 1} - 1) + L_{\mu 1}(\frac{2}{3}L_1 + L_{\mu 1} - \frac{14}{3})] + \frac{2}{3}N_f L_{\mu 1} \right\}, \end{aligned} \quad (6)$$

where $L_{\mu 1} = \log Q_1^2/\mu^2$. We make the distinction between results for physical multiplicities, which we denote \mathcal{M} , and theoretical multiplicities, which we denote \mathcal{N} . Note that

$$\mathcal{N}_{\text{inter}} + \mathcal{N}_{\text{intra}} = \mathcal{M}, \quad (7)$$

with no dependence on μ . Since μ is an entirely arbitrary parameter separating different treatments of the same phenomena, it should cancel in all physical quantities, at least to the accuracy to which we are working. However, in general it will not exactly cancel, so varying it will give a rough indication of the size of neglected terms, and hence the accuracy of our calculation. In this sense, μ plays a similar rôle to the familiar renormalization and factorization scales.

We turn now to the problem of how to resum the large logarithms of Q_1/Q_0 to all orders in α_s , while keeping exact control of the three-jet kinematics, ie. without assuming $Q_1 \ll Q$. From equations (5) and (6), it is clear that the only dependence on Q_0 is through $L_{\mu 0}$, so we aim to resum all terms of the form $\alpha_s^n L_{\mu 0}^{2n}$ (leading) and $\alpha_s^n L_{\mu 0}^{2n-1}$ (next-to-leading)[†]. We do this separately for the intrajet and interjet components.

Resumming the intrajet multiplicity

The structure seen in equation (5) continues to all orders, with the intrajet multiplicity contributing two logarithms of μ^2/Q_0^2 for every power of α_s . Therefore in resumming it, we must keep track of next-to-leading corrections to the evolution. By explicitly setting up the evolution equations, one finds that the intrajet multiplicities are identical to those in jets of the same flavour formed from a colour singlet at scale μ resolved with a transverse momentum cutoff Q_0 . Results for these quantities, which we denote $\mathcal{N}_q(Q_0, \mu)$ and $\mathcal{N}_g(Q_0, \mu)$ for quark and gluon jets, were given to next-to-leading logarithmic accuracy in [11]. We do not bother repeating the definitions here, but it is worth

*If $Q_0 > Q_1/2$, there are no large logarithms to resum, so we do not use the following procedure. In that case the leading order matrix element is perfectly sufficient on its own.

[†]We are implicitly assuming $\mu \sim Q_1$.

noting their expansions in the threshold region (formally defined as the region where $\alpha_s L_{\mu 0}^2$ is small but $L_{\mu 0}$ is still large),

$$\mathcal{N}_q(Q_0, \mu) = 1 + \bar{\alpha}_s(\mu) \left[\frac{1}{2} C_F L_{\mu 0}^2 - \frac{3}{2} C_F L_{\mu 0} \right] + \mathcal{O}(\bar{\alpha}_s^2), \quad (8)$$

$$\mathcal{N}_g(Q_0, \mu) = 1 + \bar{\alpha}_s(\mu) \left[\frac{1}{2} C_A L_{\mu 0}^2 - \frac{1}{2} b L_{\mu 0} \right] + \mathcal{O}(\bar{\alpha}_s^2), \quad (9)$$

where $b = \frac{11}{3} C_A - \frac{2N_f}{3}$. In terms of these multiplicities, the total intrajet multiplicity is simply given by

$$\mathcal{N}_{\text{intra}} = 2\mathcal{N}_q(Q_0, \mu) + \mathcal{N}_g(Q_0, \mu). \quad (10)$$

Note that μ naturally arises as the scale to use for α_s when we evaluate the fixed-order expression.

Resumming the interjet multiplicity

Physically, there are two ways that the interjet multiplicity can be increased beyond leading order: either by multiple emission from the three-jet event, or by emission from the gluon emitted at first order. Since the leading order expression is single-logarithmic in $L_{\mu 0}$, raising it to any power results in terms that are negligible to next-to-leading accuracy. This means that multiple emission is negligible, and to retain next-to-leading accuracy overall we only need to keep track of leading logarithms arising from emission from the leading order gluon.

As we already mentioned, to keep track of which jet an interjet subjet was merged into, we need to include azimuthal correlations between different emissions. Thus it seems that azimuth-dependent evolution equations are needed. However this is not the case, because to leading logarithmic accuracy, the additional partons inside the interjet subjet can be considered collinear, and so are always combined together by the jet algorithm before being combined *en masse* with one of the main jets of the event. Therefore we only need to calculate the distribution of subjets between jets to leading order in α_s , to give an expression to next-to-leading logarithmic accuracy to all orders in α_s . This is identical to the trick we used in [13], where we calculated interjet terms (in that case generated by initial-state radiation) for the number of subjets in a hadron-collision jet, despite being unable to obtain analytical results for the kinematic configurations that generated them.

In calculating the number of subjets in the interjet region, one encounters integrals of the general form

$$\mathcal{N}_{\text{inter}} \sim \int_{Q_0^2}^{\mu^2} dk_{\perp}^2 \int d\Phi \frac{d\sigma}{dk_{\perp}^2 d\Phi} \Theta(k_{\perp}, \Phi) \mathcal{N}_g(Q_0, k_{\perp}), \quad (11)$$

where k_{\perp} is the transverse momentum of the interjet gluon, Φ represents the rest of the phase-space, namely an energy fraction and an azimuth, and Θ represents the (arbitrarily complicated) phase-space constraints. This can always be manipulated into the form

$$\mathcal{N}_{\text{inter}} = A_{\text{inter}} \int_{Q_0^2}^{\mu^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \bar{\alpha}_s(k_{\perp}) \mathcal{N}_g(Q_0, k_{\perp}) + \bar{\alpha}_s(\mu) B_{\text{inter}}, \quad (12)$$

where A_{inter} and B_{inter} are arbitrarily complicated non-logarithmic functions, and we have neglected \mathcal{N}_g in the second term, which is justified to next-to-leading logarithmic

accuracy. Noting that the leading order expansion of \mathcal{N}_g is just unity, we obtain the exact result to leading order in α_s ,

$$\mathcal{N}_{\text{inter}}^{\alpha_s} = \bar{\alpha}_s(\mu) A_{\text{inter}} L_{\mu 0} + \bar{\alpha}_s(\mu) B_{\text{inter}}. \quad (13)$$

To calculate the integral retaining \mathcal{N}_g , we use the evolution equation to leading logarithmic order,

$$\mathcal{N}'_g(Q_0, \mu) = C_A \int_{Q_0^2}^{\mu^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \bar{\alpha}_s(k_{\perp}) \mathcal{N}_g(Q_0, k_{\perp}) \quad (14)$$

where the prime denotes logarithmic differentiation with respect to the second index[†]

$$\mathcal{N}'_g(Q_0, Q) \equiv Q^2 \frac{d}{dQ^2} \mathcal{N}_g(Q_0, Q). \quad (15)$$

Using this to rewrite (12), we arrive at the same result exact to both leading order in α_s , and next-to-leading order in $L_{\mu 0}$ for all orders in α_s ,

$$\mathcal{N}_{\text{inter}} = A_{\text{inter}} \frac{1}{C_A} \mathcal{N}'_g(Q_0, \mu) + \bar{\alpha}_s(\mu) B_{\text{inter}}. \quad (16)$$

The final point concerns the evaluation of A_{inter} and B_{inter} . As we have stressed, they are arbitrarily complicated in general, and not amenable to analytic evaluation. However, as in [13], it is possible to evaluate $\mathcal{N}_{\text{inter}}$ to next-to-leading logarithmic accuracy, without knowing their explicit forms. We note that the threshold expansion of \mathcal{N}'_g is

$$\mathcal{N}'_g(Q_0, \mu) = C_A \bar{\alpha}_s(\mu) L_{\mu 0} + \mathcal{O}(\bar{\alpha}_s^2 L_{\mu 0}^3), \quad (17)$$

and that B_{inter} is non-logarithmic, so the relation

$$\bar{\alpha}_s B_{\text{inter}} \frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}} = \bar{\alpha}_s B_{\text{inter}} \quad (18)$$

holds to the appropriate accuracy. Comparing equations (13) and (16), we arrive at the final answer for the interjet multiplicity in three-jet events, exact to leading order in α_s and resumming next-to-leading logarithms in $L_{\mu 0}$ to all orders in α_s ,

$$\mathcal{N}_{\text{inter}} = \mathcal{N}_{\text{inter}}^{\alpha_s} \frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}}. \quad (19)$$

Put simply, this means that the full result is found by numerically integrating the exact leading order matrix element, and multiplying the result by the parton multiplication factor,

$$\frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}}. \quad (20)$$

Explicit forms for A_{inter} and B_{inter} are never needed. It is worth stressing again, that this correctly keeps track of which jet each subjet is merged into to all orders in α_s , including azimuthal correlations. Note that the parton multiplication factor has threshold expansion

$$\frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}} = 1 + \frac{1}{6} C_A \bar{\alpha}_s(\mu) L_{\mu 0}^2 + \mathcal{O}(\bar{\alpha}_s^2), \quad (21)$$

so rises from unity with increasing $L_{\mu 0}$.

[†]Note that this is a factor of two smaller than the \mathcal{N}'_g we defined in [13].

The full multiplicity

In combining the results for the two regions, we have the opportunity to make a final embellishment that avoids the need to differentiate between the interjet and intrajet regions when numerically evaluating the leading order matrix element. This is because the intrajet region gives a contribution of the logarithmic terms (5) plus a non-logarithmic term. Multiplying non-logarithmic terms by the parton multiplication factor gives negligible next-to-next-to-leading logarithmic corrections. Therefore the full expression for the multiplicity reads

$$\begin{aligned} \mathcal{M} = & \bar{\alpha}_s(\mu) \left(\mathcal{M}^{\alpha_s} - 2 \left[\frac{1}{2} C_F L_{\mu 0}^2 - \frac{3}{2} C_F L_{\mu 0} \right] - \left[\frac{1}{2} C_A L_{\mu 0}^2 - \frac{1}{2} b L_{\mu 0} \right] \right) \frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}} \\ & + 2\mathcal{N}'_q(Q_0, \mu) + \mathcal{N}'_g(Q_0, \mu), \end{aligned} \quad (22)$$

where all the dependence on μ has been explicitly displayed.

As a final check of this formula, we show that it is independent of μ to the appropriate accuracy by taking the logarithmic derivative,

$$\begin{aligned} \mu^2 \frac{d\mathcal{M}}{d\mu^2} = & \bar{\alpha}_s(\mu) \left(-2 \left[C_F L_{\mu 0} - \frac{3}{2} C_F \right] - \left[C_A L_{\mu 0} - \frac{1}{2} b \right] \right) \frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}} \\ & + \bar{\alpha}_s(\mu) \left(\mathcal{M}^{\alpha_s} - 2 \left[\frac{1}{2} C_F L_{\mu 0}^2 - \frac{3}{2} C_F L_{\mu 0} \right] - \left[\frac{1}{2} C_A L_{\mu 0}^2 - \frac{1}{2} b L_{\mu 0} \right] \right) \\ & \times \left(\frac{\mathcal{N}''_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}} - \frac{\mathcal{N}'_g(Q_0, \mu)}{C_A \bar{\alpha}_s(\mu) L_{\mu 0}^2} \right) \\ & + 2\mathcal{N}'_q(Q_0, \mu) + \mathcal{N}'_g(Q_0, \mu). \end{aligned} \quad (23)$$

The first and third terms cancel each other exactly to leading order in α_s and, using the leading logarithmic result

$$\mathcal{N}'_q(Q_0, \mu) = \frac{C_F}{C_A} \mathcal{N}'_g(Q_0, \mu), \quad (24)$$

to next-to-leading logarithmic accuracy for all orders in α_s . The two terms in the second bracket of the second term exactly cancel to leading order in α_s , but at first sight it appears that there are non-zero terms at next-to-leading logarithmic accuracy ($\alpha_s(\mu) \times L_{\mu 0}^2 \times$ a next-to-leading logarithmic function). However, note that \mathcal{M}^{α_s} contains L_0^2 terms with the same coefficients as the $L_{\mu 0}^2$ functions, so for $Q_0 \ll \mu \sim Q_1$ the first bracket is actual single logarithmic, and the whole term is zero to next-to-leading logarithmic accuracy.

Therefore the expression (22) is indeed independent of μ , as we claimed, and gives the full result exact to leading order in α_s , resumming leading and next-to-leading logarithms to all orders in α_s , and correctly keeping track of which jet the subjects get merged into.

In figure 1a we show Eq. (22) in comparison with the data of [6] for $y_1 = 0.1$. We fix $\Lambda_{\text{QCD}} = 250$ MeV, and use the two-loop running coupling (although this is not justified at our accuracy) corresponding to $\alpha_s(M_Z) = 0.120$. We see that agreement is good down to about $y_0 = 2 \times 10^{-3}$, which corresponds to transverse momenta of about 4 GeV. Below this value, the data rise above the predictions. The barely distinguishable pairs of curves are the result of varying the separation scale μ , with the natural choice, $Q_1/2$, giving

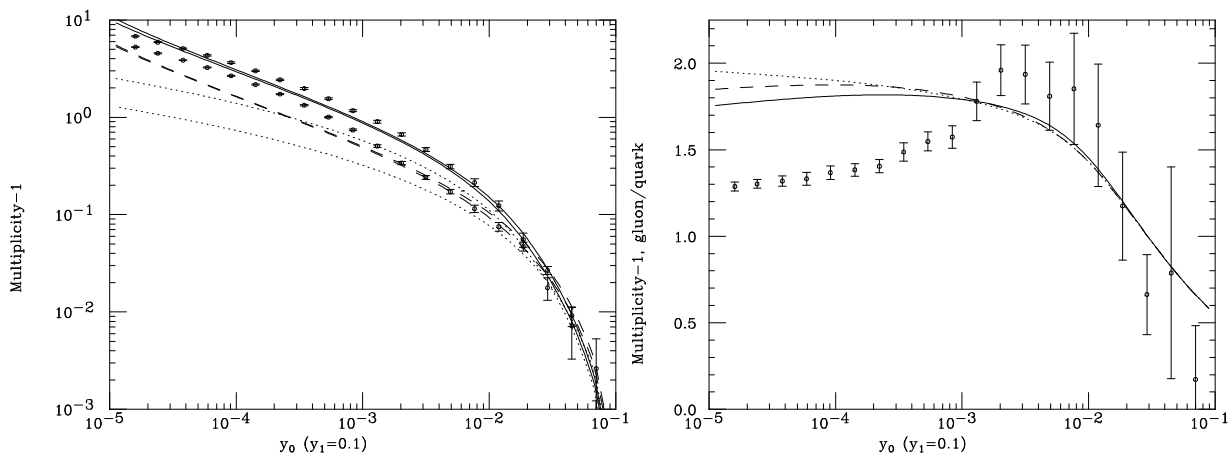


Figure 1: *The subjet multiplicity in quark and gluon jets, with jet definition scale $y_1 = 0.1$, in comparison with ALEPH data[6]. (a) Absolute values for gluon jets (solid) and quark jets (dashed) with $\mu = \sqrt{y_1}Q/2$ and $\mu = \sqrt{y_1}Q/4$. Also shown are the leading order results (dotted). (b) The ratio of gluon to quark jets, with $\mu = \sqrt{y_1}Q/2$ (solid), $\mu = \sqrt{y_1}Q/4$ (dashed) and at leading order (dotted).*

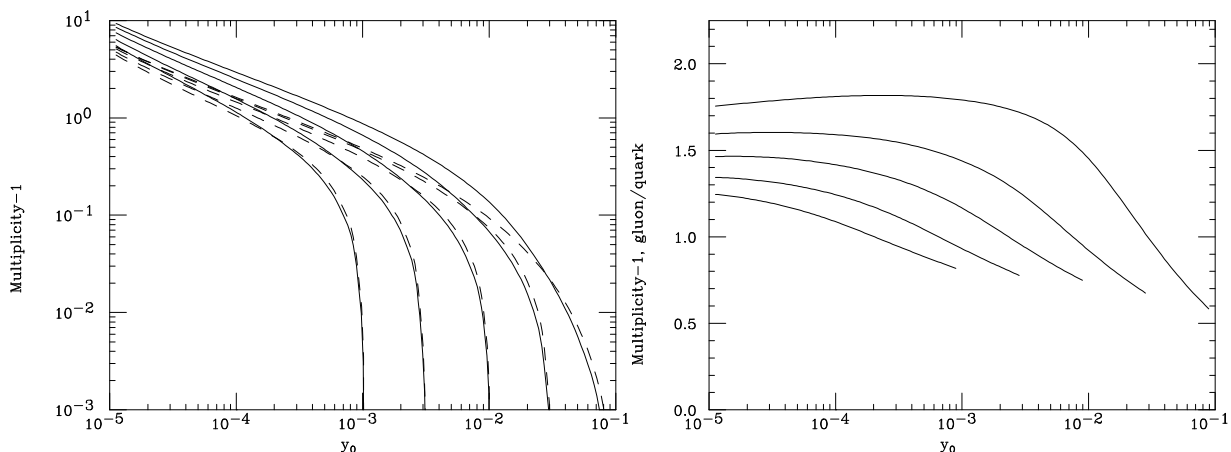


Figure 2: *As in figure 1 but for varying values of $y_1 = 0.1, 0.03, 0.01, 0.003$ and 0.001 .*

the lower curve, and $Q_1/4$ giving the upper. Also shown are the leading order curves, which fall below the data well before the full results.

In figure 1b we show the ratio of the gluon and quark multiplicities. It can be immediately seen that in the region in which the multiplicities rise above our predictions, $y_0 < 2 \times 10^{-3}$, the quark multiplicity is increased more than the gluon, suppressing the multiplicity ratio. The effect of varying the separation scale is more pronounced simply because the vertical axis is more compressed. As shown in [6], the disagreement between our results and the data can be attributed to hadronization corrections both at small y_0 and the intermediate y_0 values where the data lie slightly above our curve. In fact the parton-level predictions of the JETSET program shown there agree almost perfectly with our curves, while the hadron-level predictions agree almost perfectly with the data.

In figure 2 we show the y_1 -dependence of the results, which is considerable, although the general features remain unchanged as y_1 is varied. If a measurement of the y_1 -dependence could be made, it could be used to separate the interjet and intrajet components of multiplicity, without needing to explicitly measure the subjet directions.

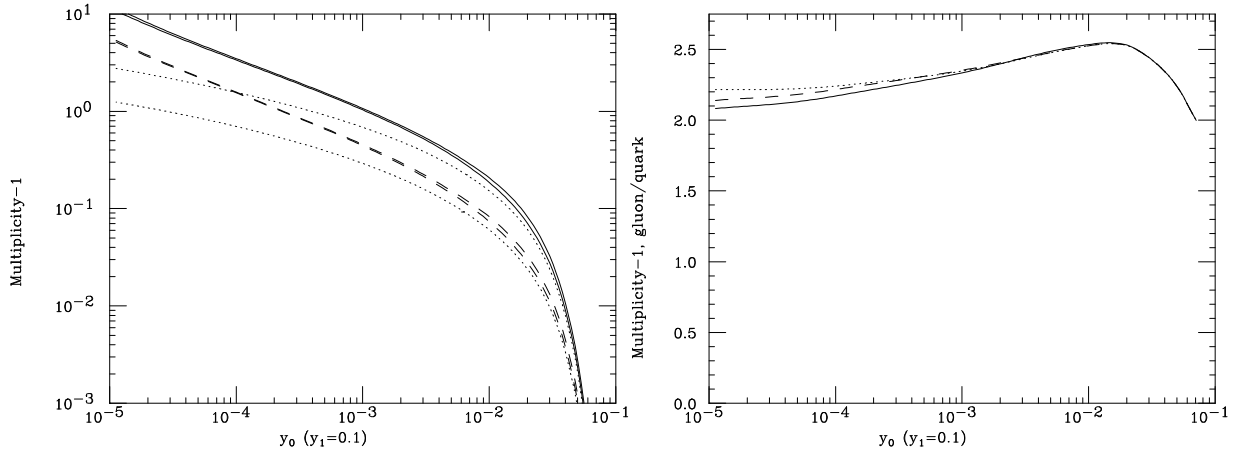


Figure 3: As in figure 1 but for jets in the energy range $30 < E_{\text{jet}}/\text{GeV} < 35$.

4 Jet Energy Dependence

In the full results there is clearly a reduction in the difference between gluon and quark jets, relative to the leading logarithmic expectation. One might suspect that at least part of this is because the gluon jet tends to be the least energetic, despite the fact that the selection $y_1 = 0.1$ requires all three jets to have fairly similar energy. This can easily be checked by binning the events according to jet energy.

Checking Eq. (22) critically, one finds that it holds equally well for the multiplicity as a function of the three-jet kinematics. Specifically, none of the resummed components depend on the three-jet kinematics, so if \mathcal{M}^{α_s} is binned in jet energy, then \mathcal{M} will also be binned correctly.

As an example, we show in figure 3 the results for jets with energy in the range $30 < E_{\text{jet}}/\text{GeV} < 35$. We see that the multiplicity ratio is indeed enhanced, particularly at larger y_0 values.

5 Summary

By combining numerical integration of the leading order matrix element with analytical resummation of large logarithmic terms, we have obtained an expression for the subjet multiplicity in 3-jet e^+e^- events (22) that is uniformly reliable for all values of y_0/y_1 , and correctly incorporates azimuthal correlations between jet and subjet directions. Where the hadronization corrections are estimated to be small the results are in good agreement with data. It is possible that modifications to the jet definition could reduce the size of these corrections, and the scale at which they begin to become important. The method used in this paper could easily be adapted to any definition in which the systematic resummation of large logarithmic terms to all orders is possible. It would also be straightforward to adapt it for any other three-jet selection or flavour-tagging method, since only the fixed-order matrix element depends on the three-jet kinematics and this is integrated numerically.

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